Even and odd phase coherent states for Hermitian phase operator theory

1996 J. Phys. A: Math. Gen. 29 895

(http://iopscience.iop.org/0305-4470/29/4/016)

J. Phys. A: Math. Gen. **29** (1996) 895–901. Printed in the UK

Even and odd phase coherent states for Hermitian phase operator theory

Le-Man Kuang† and Jiu-Yun Zhu‡

† Theoretical Physics Division, Nankai Institute of Mathematics, Tianjin 300071, People's Republic of China
and
Department of Physics and Institute of Physics, Hunan Normal University, Hunan 410006, People's Republic of China
‡ Department of Physics and Institute of Physics, Hunan Normal University, Hunan 410081, People's Republic of China

Received 18 September 1995

Abstract. Even and odd phase coherent states associated with the Hermetian phase operator theory are introduced in terms of the creation operation of the phase quanta defined in a finite-dimensional phase state space. Some mathematical and physical properties of these quantum states are studied in some detail. It is shown that the even phase coherent states together with the odd ones build an overcomplete Hilbert space. Even and odd coherent-state formalism of the Pegg–Barnett phase operator is given in terms of the projection operator in the even and odd phase coherent-state space. The number–phase uncertainty relation is investigated for these quantum states. It is shown that even and odd phase coherent states are not minimum uncertainty and intelligent states for the number and phase operators.

It is well known that quantum phase [1] is an important concept in physics, especially in quantum optics [2], which has been given a great deal of attention for a long time. In quantum mechanics, any observable should be related to a Hermitian operator. The problem of defining a Hermitian phase operator is an old one. Susskind and Glogower (SG) [3] established the exponential phase operators. Since then the SG phase operators have applied to a variety of problems in quantum optics [4]. In particular, Shapiro and Shepard [5] proposed coherent and squeezed phase states which can be used to study quantum phase measurements [6]. Unfortunately, the SG phase theory suffers from the fact that a Hermitian phase operator can not be constructed since the energy spectrum of a harmonic oscillator is restricted from below.

A very important development in this field has been made by Pegg and Barnett (PB) [7–9]. They have defined a Hermitian phase operator in a finite-dimensional but arbitrarily large Hilbert space. Since the natural description of the electromagnetic field means the use of infinite-dimensional Hilbert space, one must take the infinite-dimensional limit at the end of expectation-value calculations performed in the finite-dimensional space. Authors in [10–13] have adopted the PB phase formalism to study properties of coherent states and a harmonic oscillator in the finite-dimensional space. In our previous paper [14], we studied the even and odd coherent state in the finite-dimensional number-state space. Making use of the factorizable property of the PB Hermitian phase operator, in [15, 16] we have constructed

0305-4470/96/040895+07$19.50

phase coherent states and obtained the phase coherent-state formalism of the Hermitian phase operator. In this paper, we intend to introduce even and odd phase coherent states associated with the PB Hermitian phase theory and study some properties of these quantum states.

Based on a finite-dimensional number-states space $\sum_{2s+1} = \{|0\rangle, |1\rangle, \ldots, |2s\rangle\}$, where s is a positive integer, a Hermitian phase operator can be defined in terms of the projection operator of the $(2s+1)$-dimensional phase-state space $\sum_{2s+1} = \{|\theta_0\rangle, |\theta_1\rangle, \ldots, |\theta_{2s}\rangle\}$ in this form:

$$\hat{\Phi}_\theta = \sum_{m=0}^{2s} \theta_m |\theta_m\rangle\langle\theta_m| \tag{1}$$

where the phase eigenvalue $\theta_m = \theta_o + \frac{2\pi m}{2s+1}$ with $m = 0, 1, \ldots, 2s$, θ_o is an arbitrary reference phase. For simplicity, we set $\theta_o = 0$ throughout the paper. The phase states $|\theta_m\rangle$ are defined by

$$|\theta_m\rangle = \frac{1}{\sqrt{2s+1}} \sum_{n=0}^{2s} \exp(\mathrm{i}\theta_m n)|n\rangle \tag{2}$$

which forms an orthonormal completeness Hilbert space with properties

$$\langle\theta_m|\theta_n\rangle = \delta_{m,n} \qquad \sum_{m=0}^{2s} |\theta_m\rangle\langle\theta_m| = 1\,. \tag{3}$$

Making use of the projection operator of the phase-state space, one can construct the annihilation and creation operators of the phase quanta as follows:

$$\hat{p} = \sum_{m=1}^{2s} \sqrt{\theta_m}|\theta_{m-1}\rangle\langle\theta_m| \qquad \hat{p}^\dagger = \sum_{m=1}^{2s} \sqrt{\theta_m}|\theta_m\rangle\langle\theta_{m-1}| \tag{4}$$

which act on the $(2s+1)$-dimensional phase-state space $\sum'_{2s+1}$ in the following form:

$$\hat{p}|\theta_m\rangle = \sqrt{\theta_m}|\theta_{m-1}\rangle \qquad \hat{p}|0\rangle_p = 0 \tag{5}$$

$$\hat{p}^\dagger|\theta_m\rangle = \sqrt{\theta_{m+1}}|\theta_{m+1}\rangle \qquad \hat{p}^\dagger|\theta_{2s}\rangle = 0 \tag{6}$$

where the phase vacuum is defined as $|0\rangle_p \equiv |\theta_o = 0\rangle$ which has the number-state representation

$$|0\rangle_p = \frac{1}{2s+1} \sum_{n=0}^{2s} |n\rangle\,. \tag{7}$$

The PB Hermitian phase operator is then factorized as

$$\hat{\Phi}_\theta = \hat{p}^\dagger \hat{p}\,. \tag{8}$$

These operators $\hat{p}$, $\hat{p}^\dagger$ and $\hat{\Phi}_\theta$ satisfy the following commutation relations:

$$[\hat{p}, \hat{p}^\dagger] = \frac{2\pi}{2s+1} - 2\pi|\theta_{2s}\rangle\langle\theta_{2s}| \tag{9}$$

$$[\hat{\Phi}_\theta, \hat{p}] = -\hat{p} \qquad [\hat{\Phi}_\theta, \hat{p}^\dagger] = \hat{p}^\dagger\,. \tag{10}$$

Like even and odd coherent states in a finite-dimensional number-state [14], we introduce even and odd phase coherent states in the phase-state space $\sum'_{2s+1}$ in the following form:

$$|Z\rangle_e \equiv N_e(Z)\cosh(Z\hat{p}^+)|0\rangle_p = N_e(Z) \sum_{n=0}^{s} \frac{\tilde{Z}^{2n}}{\sqrt{(2n)!}}|\theta_{2n}\rangle \tag{11}$$

$$|Z\rangle_o \equiv N_o(Z)\sinh(Z\hat{p}^+)|0\rangle_p = N_o(Z)\sum_{n=0}^{s-1}\frac{\tilde{Z}^{2n+1}}{\sqrt{(2n+1)!}}|\theta_{2n+1}\rangle \tag{12}$$

where we have used (6), Z is an arbitrary complex number and $\tilde{Z} = \sqrt{\frac{2\pi}{2s+1}}Z$.

The normalization constants $N_e(Z)$ and $N_o(Z)$ can be obtained from the normalization conditions:

$$_e\langle Z|Z\rangle_e = 1 \qquad _o\langle Z|Z\rangle_o = 1 \tag{13}$$

which leads to

$$N_e(Z) = \cosh_s^{-1/2}(|\tilde{Z}|^2) \qquad N_o(Z) = \sinh_s^{-1/2}(|\tilde{Z}|^2) \tag{14}$$

where we have used the following polynomial functions:

$$\sinh_s x \equiv \sum_{n=0}^{s-1}\frac{x^{2n+1}}{(2n+1)!} \qquad \cosh_s x \equiv \sum_{n=0}^{s}\frac{x^{2n}}{(2n)!}\,. \tag{15}$$

Taking into account (2), we can obtain the number-state representations of the even and odd phase coherent states as follows:

$$|Z\rangle_e = \frac{N_e(Z)}{\sqrt{2s+1}}\sum_{m=0}^{s}\sum_{n=0}^{2s}\frac{\tilde{Z}^{2m}}{\sqrt{(2m)!}}\exp(\mathrm{i}\theta_{2m}n)|n\rangle \tag{16}$$

$$|Z\rangle_o = \frac{N_o(Z)}{\sqrt{2s+1}}\sum_{m=0}^{s-1}\sum_{n=0}^{2s}\frac{\tilde{Z}^{2m+1}}{\sqrt{(2m+1)!}}\exp(\mathrm{i}\theta_{2m+1}n)|n\rangle\,. \tag{17}$$

It is straightforward to show that the even (or odd) phase coherent states cannot form separately a complete set. However, the even phase coherent states together with the odd ones build an overcomplete Hilbert space. Their completeness relation takes this form:

$$\int \mathrm{d}^2Z[\sigma_e(Z)|Z\rangle_{ee}\langle Z| + \sigma_o(Z)|Z\rangle_{oo}\langle Z|] = \sum_{m=0}^{2s}|\theta_m\rangle\langle\theta_m| = 1 \tag{18}$$

where $\mathrm{d}^2Z = |Z||\mathrm{d}|Z|\,\mathrm{d}\phi$ with $Z = |Z|\mathrm{e}^{\mathrm{i}\phi}$, the two weight functions are given by

$$\sigma_e(Z) = \frac{1}{\pi}\cosh_s\left(\frac{2\pi}{2s+1}|Z|^2\right) \qquad \sigma_o(Z) = \frac{1}{\pi}\sinh_s\left(\frac{2\pi}{2s+1}|Z|^2\right). \tag{19}$$

These even and odd phase coherent states have the following orthogonal relations:

$$_e\langle Z|Z'\rangle_e = N_e(Z)N_e(Z')\cosh_s(\tilde{Z}^*\tilde{Z}') \tag{20}$$

$$_o\langle Z|Z'\rangle_o = N_o(Z)N_o(Z')\sinh_s(\tilde{Z}^*\tilde{Z}') \tag{21}$$

$$_o\langle Z|Z'\rangle_e = 0 \tag{22}$$

where $\tilde{Z}^*$ is the complex conjugation of $\tilde{Z}$.

From (16) and (17), we obtain the probability distributions of the even and odd phase coherent states in the phase-state space,

$$P(\theta_m\|Z)_e \equiv |\langle\theta_m|Z\rangle_e|^2 = \frac{1}{(2m)!}\left(\frac{2\pi|Z|^2}{2s+1}\right)^{2m}\cosh_s^{-1}\left(\frac{2\pi|Z|^2}{2s+1}\right) \tag{23}$$

$$P(\theta_m\|Z)_o \equiv |\langle\theta_m|Z\rangle_o|^2 = \frac{1}{(2m+1)!}\left(\frac{2\pi|Z|^2}{2s+1}\right)^{2m+1}\sinh_s^{-1}\left(\frac{2\pi|Z|^2}{2s+1}\right). \tag{24}$$

Then, the phase variances associated with the above distributions are given by, respectively,

$$\langle(\Delta\hat{\Phi}_\theta)^2\rangle_e = \frac{1}{2}\cosh_s^{-2}\left(\frac{2\pi|Z|^2}{2s+1}\right)\sum_{n\neq m}^{s}\frac{(\theta_{2n}-\theta_{2m})^2}{(2n)!(2m)!}\left(\frac{2\pi|Z|^2}{2s+1}\right)^{2(n+m)} \tag{25}$$

$$\langle(\Delta\hat{\Phi}_\theta)^2\rangle_o = \frac{1}{2}\sinh_s^{-2}\left(\frac{2\pi|Z|^2}{2s+1}\right)\sum_{n\neq m}^{s}\frac{(\theta_{2n+1}-\theta_{2m+1})^2}{(2n+1)!(2m+1)!}\left(\frac{2\pi|Z|^2}{2s+1}\right)^{2(n+m)+2}. \tag{26}$$

One can also obtain the probability distributions of the even and odd phase coherent states in the number-state representation:

$$\begin{aligned} P(n\|Z\rangle_e) &\equiv |\langle n|Z\rangle_e|^2 \\ &= \frac{1}{2s+1}\left|\sum_{m=0}^{s}\sqrt{\frac{2\pi}{(2s+1)(2m)!}}Z^{2m}\exp\left(\mathrm{i}\frac{4\pi mn}{2s+1}\right)\right|^2\cosh_s^{-1}\left(\frac{2\pi|Z|^2}{2s+1}\right) \end{aligned} \tag{27}$$

$$\begin{aligned} P(n\|Z\rangle_o) &\equiv |\langle n|Z\rangle_o|^2 \\ &= \frac{1}{2s+1}\left|\sum_{m=0}^{s-1}\sqrt{\frac{2\pi}{(2s+1)(2m+1)!}}Z^{2m+1}\exp\left(\mathrm{i}\frac{2\pi(2m+1)n}{2s+1}\right)\right|^2 \\ &\quad\times\sinh_s^{-1}\left(\frac{2\pi|Z|^2}{2s+1}\right). \end{aligned} \tag{28}$$

On the basis of the completeness relation (18), we can expand a phase state $|\theta_m\rangle$ in terms of the even and odd phase coherent states as

$$|\theta_m\rangle = \frac{1}{\pi\sqrt{m!}}\int \mathrm{d}^2Z\left[\cosh_s^{1/2}\left(\frac{2\pi|Z|^2}{2s+1}\right)|Z\rangle_e + \sinh_s^{1/2}\left(\frac{2\pi|Z|^2}{2s+1}\right)|Z\rangle_o\right]. \tag{29}$$

Any state in the phase-state space must possess the following expansion:

$$|\psi\rangle = \sum_{m=0}^{2s} C_m|\theta_m\rangle \tag{30}$$

where $\sum_{n=0}^{2s}|C_m|^2 = 1$.

In order to expand the arbitrary state in terms of the even and odd phase coherent states, we must use the completeness relations (18). Substituting (29) into (30), we obtain the following expansion in the even and odd coherent-state representation:

$$|\psi\rangle = \frac{1}{\pi}\sum_{m=0}^{2s}\frac{C_m}{\sqrt{m!}}\int \mathrm{d}^2Z\left[\cosh_s^{1/2}\left(\frac{2\pi|Z|^2}{2s+1}\right)|Z\rangle_e + \sinh_s^{1/2}\left(\frac{2\pi|Z|^2}{2s+1}\right)|Z\rangle_o\right]. \tag{31}$$

We can also expand as n quantum state in the finite-dimensional space $\sum_{2s+1}$ in the form:

$$\begin{aligned} |n\rangle = \frac{1}{\pi\sqrt{2s+1}}\int \mathrm{d}^2Z\Bigg\{&\sum_{m=0}^{s}\frac{1}{\sqrt{(2m)!}}\exp\left(\mathrm{i}\frac{4\pi mn}{2s+1}\right)\left(\frac{2\pi Z^*}{2s+1}\right)^{2m}\cosh_s^{1/2}\left(\frac{2\pi|Z|^2}{2s+1}\right)|Z\rangle_e \\ &+\sum_{m=0}^{s-1}\frac{1}{\sqrt{(2m+1)!}}\exp\left[\mathrm{i}\frac{2\pi(2m+1)n}{2s+1}\right]\left(\frac{2\pi Z^*}{2s+1}\right)^{2m+1} \\ &\times\sinh_s^{1/2}\left(\frac{2\pi|Z|^2}{2s+1}\right)|Z\rangle_o\Bigg\}. \end{aligned} \tag{32}$$

An arbitrary state in the number-state space $\sum_{2s+1}$ must possess this form:

$$|\Phi\rangle = \sum_{n=0}^{2s} D_n|n\rangle \tag{33}$$

where $\sum_{n=0}^{2s}|D_n|^2 = 1$. From (32) and (33) it is straightforward to get the even and odd phase coherent-state expansions of $|\Phi\rangle$:

$$|\Phi\rangle = \frac{1}{\pi\sqrt{2s+1}}\sum_{n=0}^{2s}\int d^2Z\Bigg\{\sum_{m=0}^{s}\frac{1}{\sqrt{(2m)!}}\exp\left(i\frac{4\pi mn}{2s+1}\right)\left(\frac{2\pi Z^*}{2s+1}\right)^{2m} \times\cosh_s^{1/2}\left(\frac{2\pi|Z|^2}{2s+1}\right)|Z\rangle_e + \sum_{m=0}^{s-1}\frac{1}{\sqrt{(2m+1)!}}\exp\left[i\frac{2\pi(2m+1)}{2s+1}\right] \times\left(\frac{2\pi Z^*}{2s+1}\right)^{2m+1}\sinh_s^{1/2}\left(\frac{2\pi|Z|^2}{2s+1}\right)|Z\rangle_o\Bigg\}. \tag{34}$$

It is easy to prove that there is the following relation between the even and odd phase coherent states and the phase coherent states [15]:

$$|Z\rangle = \frac{\sqrt{2}}{2}\left\{\left[1+\exp_{2s}\left(\frac{2\pi|Z|^2}{2s+1}\right)\right]^{1/2}|Z\rangle_e + \left[1-\exp_{2s}\left(-\frac{2\pi|Z|^2}{2s+1}\right)\right]^{1/2}|Z\rangle_o\right\} \tag{35}$$

where $\exp_{2s} x \equiv \sum_{n=0}^{2s} x^n/n!$ and the phase coherent state is defined by

$$|Z\rangle = \exp_{2s}^{-\frac{1}{2}}\left(\frac{2\pi|Z|^2}{2s+1}\right)\sum_{n=0}^{2s}\frac{1}{\sqrt{n!}}\left(\sqrt{\frac{2\pi}{2s+1}}Z\right)^n|\theta_n\rangle . \tag{36}$$

Using the completeness relation of the even and odd coherent state, through a tedious calculation, we arrive at the even and odd coherent-state representation of the Hermitian phase operator with the following result:

$$\hat{\Phi}_\theta = \int d^2Z g(Z)[f_e^2(Z)|Z\rangle_{ee}\langle Z| + f_o^2(Z)|Z\rangle_{oo}\langle Z| + f_e(Z)f_o(Z)|Z\rangle_{eo}\langle Z| + f_o(Z)f_e(Z)|Z\rangle_{oe}\langle Z|] \tag{37}$$

where

$$f_e(Z) = \frac{\sqrt{2}}{2}\left[1+\exp_{2s}\left(\frac{2\pi|Z|^2}{2s+1}\right)\right]^{1/2} \qquad f_o(Z) = \frac{\sqrt{2}}{2}\left[1-\exp_{2s}\left(\frac{2\pi|Z|^2}{2s+1}\right)\right]^{1/2} \tag{38}$$

$$g(Z) = -\frac{4\pi}{2s+1}\left(1-\frac{2\pi|Z|^2}{2s+1}\right)\exp_{2s}^{1/2}\left(\frac{2\pi|Z|^2}{2s+1}\right). \tag{39}$$

It follows from (37) that

$$\hat{\Phi}_\theta|Z\rangle_e = \int d^2Z'\Phi_e(Z,Z')|Z'\rangle_e \tag{40}$$

$$\hat{\Phi}_\theta|Z\rangle_o = \int d^2Z'\Phi_o(Z,Z')|Z'\rangle_o \tag{41}$$

where

$$\Phi_e(Z,Z') = N_e(Z)N_e(Z')g(Z')f_e(Z')[f_e(Z')+f_o(Z')]\cosh_s\left(\frac{2\pi ZZ'^*}{2s+1}\right) \tag{42}$$

$$\Phi_o(Z,Z') = N_o(Z)N_o(Z')g(Z')f_o(Z')[f_e(Z')+f_o(Z')]\sinh_s\left(\frac{2\pi ZZ'^*}{2s+1}\right). \tag{43}$$

We can also obtain the even and odd coherent-state representation of the exponential phase operator,

$$\exp(\pm i\hat{\Phi}_\theta) = \int d^2Z h_\pm(Z)[f_e^2(Z)|\rangle_{ee}\langle Z| + f_o^2(Z)|Z\rangle_{oo}\langle Z| + f_e(Z)f_o(Z)|Z\rangle_{eo}\langle Z| + f_e(Z)f_o(Z)|Z\rangle_{oe}\langle Z|] \tag{44}$$

where

$$h_\pm(Z) = \frac{2}{2s+1}\exp\left(\mp i\frac{2\pi}{2s+1}\right)\exp_{2s}\left(\frac{2\pi|Z|^2}{2s+1}\right)\exp\left[-\frac{2\pi|Z|^2}{2s+1}\exp\left(\mp i\frac{2\pi}{2s+1}\right)\right]. \tag{45}$$

Obviously, like the phase-state representation and the phase coherent-state representation [15] of the Hermitian phase operator, the even and odd phase coherent-state representation can also be used to study phase properties of the single-mode field.

Finally, we turn to discuss the number–phase uncertainty relation for the even and odd phase coherent states. The phase variances in these states have been given by (25) and (26). Through a tedious calculation, we get the number variances in these states:

$$\langle(\Delta\hat{N})^2\rangle_e = \frac{s(s+1)}{3} + \cosh_s^{-1}(|\tilde{Z}|^2)\sum_{n>m}^{s}\frac{|Z|^{2(n+m)}}{\sqrt{(2n)!(2m)!}}\frac{\cos(\Delta_{nm}/2-\alpha_{nm})}{\sin^2(\Delta_{nm}/2)} - \cosh_s^{-1}(|\tilde{Z}|^2)\left[\sum_{n>m}^{s}\frac{|\tilde{Z}|^{2(n+m)}}{\sqrt{(2n)!(2m)!}}\frac{\sin(\Delta_{nm}/2-\alpha_{nm})}{\sin(\Delta_{nm}/2)}\right]^2 \tag{46}$$

$$\langle(\Delta\hat{N})^2\rangle_o = \frac{s(s+1)}{3} + \sinh_s^{-1}(|\tilde{Z}|^2)\sum_{n>m}^{s-1}\frac{|Z|^{2(n+m+1)}}{\sqrt{(2n+1)!(2m+1)!}}\frac{\cos(\Delta_{nm}/2-\alpha_{nm})}{\sin^2(\Delta_{nm}/2)} - \sinh_s^{-1}(|\tilde{Z}|^2)\left[\sum_{n>m}^{s-1}\frac{|\tilde{Z}|^{2(n+m+1)}}{\sqrt{(2n+1)!(2m+1)!}}\frac{\sin(\Delta_{nm}/2-\alpha_{nm})}{\sin(\Delta_{nm}/2)}\right]^2 \tag{47}$$

where $\Delta_{nm} = \theta_{2n} - \theta_{2m}$ and $\alpha_{nm} = 2(n-m)\phi$. In the derivation of the above expressions we have used the following identities:

$$\frac{e^{i\alpha}}{1-e^{i\Delta}} + \frac{e^{-i\alpha}}{1-e^{-i\Delta}} = \frac{\sin(\Delta/2-\alpha)}{\sin\Delta/2} \tag{48}$$

$$\frac{\cos\alpha}{\sin^2\Delta/2} - \frac{\sin(\Delta/2-\alpha)}{\sin\Delta/2} = \frac{\cos\Delta/2\cos(\Delta/2-\alpha)}{\sin^2\Delta z/2}\,. \tag{49}$$

The expectation values of the commutator $[\hat{\Phi}_\theta, \hat{N}]$ for the even and odd phase coherent states are given by

$$\langle[\hat{\Phi}_\theta, \hat{N}]\rangle_e = i\cosh_s^{-1}(|\tilde{Z}|^2)\sum_{n>m}^{s}\frac{|\tilde{Z}|^{2(n+m)}\Delta_{nm}}{\sqrt{(2n)!(2m)!}}\frac{\cos(\Delta_{nm}/2-\alpha_{nm})}{\sin(\Delta_{nm}/2)} \tag{50}$$

$$\langle[\hat{\Phi}_\theta, \hat{N}]\rangle_o = i\sinh_s^{-1}(|\tilde{Z}|^2)\sum_{n>m}^{s-1}\frac{|\tilde{Z}|^{2(n+m+1)}\Delta_{nm}}{\sqrt{(2n+1)!(2m+1)!}}\frac{\cos(\Delta_{nm}/2-\alpha_{nm})}{\sin(\Delta_{nm}/2)} \tag{51}$$

where we have used the following formula:

$$\sum_{k=0}^{2s} k e^{ik\Delta_{nm}} = -\frac{2s+1}{1-e^{i\Delta_{nm}}} \qquad n \neq m\,. \tag{52}$$

From (25), (26), (46), (47), (50) and (51) we find that

$$\langle(\Delta\hat{\Phi}_\theta)^2\rangle_e\langle(\Delta\hat{N})^2\rangle_e \neq \tfrac{1}{4}|\langle[\hat{\Phi}_\theta, \hat{N}]\rangle_e|^2 \tag{53}$$

$$\langle(\Delta\hat{\Phi}_\theta)^2\rangle_o\langle(\Delta\hat{N})^2\rangle_o \neq \tfrac{1}{4}|\langle[\hat{\Phi}_\theta, \hat{N}]\rangle_o|^2 \tag{54}$$

which indicate that the even and odd phase coherent states are not the minimum uncertainty states and intelligent states for the number and phase operators.

In conclusion, we have constructed the even and odd phase coherent states associated with the PB Hermitian phase theory and discussed some properties of these states. We have shown that the Hermitian phase operator can be expressed in terms of the projection operator in the even and odd phase coherent-state space which can be used to investigate phase properties of the electromagnetic field. We have also studied the number–phase uncertainty relation for the even and odd phase coherent states. It is interesting to further investigate non-classical behaviours of these quantum states and their applications to quantum phase measurements. This will be discussed elsewhere.

Acknowledgment

The research was supported by the National Natural Science Foundation of China.

References

[1] Nieto M M 1994 *Phys. Scr.* T **48** 5
Barnett S M and Dalton B J 1993 *Phys. Scr.* T **48** 13
[2] Lynch R 1995 *Phys. Rep.* **256** 367
[3] Susskind L and Glogower J 1964 *Physics* **1** 49
[4] Carruthers P and Nieto M M 1968 *Rev. Mod. Phys.* **40** 411
[5] Shapiro J H and Shepard S R 1991 *Phys. Rev.* A **43** 3795
Shapiro J H, Shepard S R and Wong N C 1989 *Phys. Rev. Lett.* **62** 2377
[6] Noh J W, Fougeres A and Mandel L 1991 *Phys. Rev. Lett.* **67** 1426
[7] Pegg D T and Barnett S M 1988 *Europhys. Lett.* **6** 483
[8] Pegg D T and Barnett S M 1989 *Phys. Rev.* A **39** 1665
[9] Barnett S M and Pegg D T 1989 *J. Mod. Opt.* **36** 7
[10] Bužek V, Wilson A D, Knight P L and Lai W K 1992 *Phys. Rev.* A **45** 8079
[11] Kuang L M, Wang F B and Zhou Y G 1994 *J. Mod. Opt.* **41** 1307
[12] Miranowicz A, Piatek K and Tanas R 1994 *Phys. Rev* A **50** 3423
[13] Kuang L M, Wang F B and Zhou Y G 1993 *Phys. Lett.* **183A** 1
[14] Zhu J Y and Kuang L M 1994 *Phys. Lett.* **193A** 227
[15] Kuang L M and Chen X 1994 *Phys. Rev.* A **50** 4228
[16] Kuang L M and Chen X 1994 *Phys. Lett.* **186A** 8